Characteristics of S-jump on Roughened Bed Stilling Basin

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ABSTRACT

Formation of hydraulic jump is necessary for dissipation of the excess kinetic energy downstream of spillways and gates. Hydraulic jump stilling basins of sudden expansion are one of the energy dissipater structures which are used where the available tail water depth is less than the required sequent depths. In the present study to further reduce the hydraulic jump sequent depths ratio, a roughened bed type of stilling basins of sudden expansion is experimentally investigated. Four different expansion ratios of 0.33, 0.5, 0.67 and 1 were studied under wide range of flow conditions, Froude numbers between 2 and 10. Results of the present study show that sequent depths ratio is 16-20 percent lower and the jump efficiency can increase 20 percent in comparison to the classical jump.

Keywords

Hydraulic jump, Abrupt expansion, Stilling basins, Roughened bed

1. Introduction

Hydraulic jump stilling basins are well-known energy dissipater structures downstream of weirs, gates, sluiceways, etc. where the available tail water depth is low, so that a classical jump will not form. Even with the aid of deepening the basin floor, a lateral sudden expansion of the basin width is a practical feasible solution. Jump in expansion channel is asymmetric when the expansion rate (B) or the ratio of upstream to downstream channel widths is higher than 0.71 to 0.83 (Bremen and Hager, 1993). Hydraulic jump in abrupt expansion is described, as all other hydraulic jump basins, by the approaching flow conditions which is the inflow Froude number (F₁), inflow depth (y₁) and toe position (x₁). In hydraulic jump in abrupt expansion, tail water is a basic variable. Based on the toe position of the jump in abruptly expanded channel, four types of jumps may form, namely (Fig.1):

1. Classical jump, which is formed upstream of the expansion section. This jump is similar to the jumps that form in prismatic channels.
2. T-jump in which the toe of the jump is formed upstream of the expansion section. In this type of jump, the roller length is formed in the upstream section. T-jumps occur partly in the downstream and partly in the upstream of the expansion section. Depending on the toe position, this jump may be either symmetric or asymmetric.
3. S-jump or spatial jump which the toe is located at the expansion section. This jump is asymmetric and don’t have a specific direction.

4. R-jump or repelled jump which is formed in the downstream channel. The toe position is distant away downstream of the abrupt expansion (Bremen and Hager 1993).

Among the above mentioned types of jumps in abruptly expanded channels, the R-jump requires the lowest tail water depth. If the tail water increases it breaks down in such a way that the jump toe is shifted toward the expansion section and S-jump is formed. More increase in tail water depth, the toe of the jump moves further to the upstream expansion section and T-jump forms. By increasing the tail water, the jump is shifted upstream and will form completely at the upstream expansion section. Herbrand (1973) was first who studied S-jump in smooth expanding channels and found a simple relation for calculating the sequent depths ratio as:

$$D_B = D^* \cdot \sqrt{B}$$

Where $D_B$ is the sequent depths ratio in smooth bed and expanding section, $D^* = \frac{y_2}{y_1}$ is the sequent depths ratio in classical jump which can be calculate from the well-known Belanger’s equation and $B=b_1/b_2$ is the ratio of upstream to downstream channel widths.

Matin et al. (1997) investigated jumps in expanding channels with smooth bed. They applied momentum equation for the case of S-jump and developed the following equation:

$$D_B^2 - D_{BR}^2 \left[ B2F_{ij}^2 + 1 \right] + 2F_{ij}^2 B^2 = 0$$

Using Eq. 2 they developed the following relation for the sequent depths ratio:

$$D_B = \frac{1}{2} \left( -1 + \sqrt{1 + 8E_i^2} \right), \quad E_i^2 = \frac{F_{ij}^2}{K_s}$$

The equation is applicable for all types of hydraulic jumps in abrupt expanding channels and all expanding ratios. In the above equation, $E$ is a modified Froude number, $K_s$ is an assumed parameter which accounts for the effect of abrupt expansion of the channel on the jump depth and is defined as:

$$K_s = \frac{(1-D_B)}{B(B-D_B)}$$

Matin et al. (1997) also developed an equation based on the experimental works to express $K_s$ for the jump whose toe is located at the expansion section. The equation is as follows:

$$K_s = 1 + 4.243 \left[ \log \left( \frac{I}{B} \right) \right] \cdot \log F_{ij}$$

Fig. 1- Different types of hydraulic jumps in sudden expansion section - a) R-jump, b) S-jump, c) T-jump, and d) classical jump
In Eq. (4) if B=1 (no expansion), then $K_s=1$, $E_1=F_{r_1}$ and Eq. (2) reduces the Belanger equation for classical jump. Alhamid (2004) studied S-jump in abrupt expansion with $B=0.33$, 0.5 and 0.67, and a best fit relation for $K_s$ was found as follows:

$$K_s = 1\left[I + 0.25 \ln\left(I + \ln(1 + F_{r_1})\right)\right]$$  \hspace{1cm} (6)

In which all variables were defined previously. Chandra and Lal (1978) investigated jumps in abruptly expanded channels and introduced a sensitivity parameter for the toe displacement due to tail water variation (Bremen and Hager 1993).

$$\psi = \frac{D^* - D_B}{D^* - I} , \quad 0 < \psi < I$$  \hspace{1cm} (7)

Bremen and Hager (1993) attributed a new relation for $\psi$ in T-jumps for abrupt expanding ratios of $B=0.33$, 0.5 and 0.71 in the following form:

$$\psi = \left[I - \sqrt{B}\right]\left[I - \tgh(1.9X)\right] , \quad X_i = \frac{x_i}{L_i}$$  \hspace{1cm} (8)

The quantity $\psi$ depends on the toe position $x_1$. $\psi$ is equal to zero for $B=1$ for the jump occurring in a prismatic channel or entirely in the approaching channel. The maximum of $\psi=1$ occurs for very small $B$ when the toe is located at the expansion section and S-jump occurs. Then, if the jump in abrupt expansion section is S-jump, $\psi = \left[I - \sqrt{B}\right]$. $\psi$ for any expansion ratio is a constant value. Also they showed that the efficiency of classical jump can be calculated using Eq. (9):

$$\eta^* = \left[I - \sqrt{\frac{F_{r_1}}{F_s}}\right]^2$$  \hspace{1cm} (9)

Alhamid (2004) investigated S-jump in the abrupt expansions with $B=0.33$, 0.5 and 0.67. Then they showed that the efficiency on smooth bed in this type of jump is higher than the efficiency of the classical jump. Eq. (10) was developed by Alhamid (2004):

$$\eta = \eta^* \left[I - \frac{5.162 \ln B}{F_{r_1}^{1.774}}\right]$$  \hspace{1cm} (10)

For the case of jump on roughened bed, Rajaratnam (1968) was the first who showed that rough elements can create high turbulent flow and larger eddies in which the result would be occurrence of the jump with a less sequent depths ratio required. He stated that if the shear force exerted by the roughness is shown as $F_s = \frac{1}{2} \varepsilon b_1 y_1^2$, in which $\varepsilon$ is shear force coefficient and $b_1$ and $y_1$ are the basin width and flow depth, respectively, then $\varepsilon$ can increase due to the presence of roughness with the incoming Froude number according to the following equation.

$$\varepsilon = 0.0082\left(F_{r_1}^2 - I\right)^{0.335}$$  \hspace{1cm} (11)

Izadjoo and Shafai Bejestan (2007) used trapezoidal shape corrugated bed in their study. Their results showed that shear stress in roughened bed is ten times greater than the shear stress on smooth bed. For the sequent depths ratio and shear force coefficient they presented the following equations:

$$D_r = 1.047 F_{r_1} - 0.5902$$  \hspace{1cm} (12)

$$\varepsilon = 0.058 F_{r_1}^{1.04}$$  \hspace{1cm} (13)

Study of Shafai Bejestan and Neisi (2009) also showed that putting lozenge shape rough element on the bed of stilling basin can increase the performance of the basin. Literature review has shown that S-jump in abruptly expanded channel has the
highest efficiency in dissipating energy and requires a less tail water depth comparing to the classical jump. On the other hand, it has been shown that the roughness can reduce the sequence depth and also increase the efficiency of classical jump. Therefore, a combination of sudden expansion and roughened bed stilling basin can decrease the sequent depths ratio even more which is the main goal of the present study.

2. Theoretical expression

Fig. 2 shows a sketch of S-jump in rough bed channels.

Considering Fig. 2, the sequent depths may be computed by the application of conventional momentum equation. Assuming a) uniform velocity distribution and hydrostatic pressure at both sections 1 and 2, b) the effect of turbulence and air entrainment is negligible, c) the effect of wall friction is negligible then the momentum equation for the control volume between sections 1 and 2 of Fig. 2 can be written as follows:

\[
F_1 + F_e - F_2 - F_f = \frac{\gamma}{g} \cdot Q \cdot (\beta_2 V_2 - \beta_1 V_1)
\] (14)

Where \(F_1\) and \(F_2\) are the hydrostatic pressure forces at upstream and downstream of the jump, respectively, \(F_f\) is the shear force that for the roughened bed can be obtained from \(F_f = \frac{1}{2} \gamma \varepsilon b_1 y_1^2\); in which \(\varepsilon\) is the shear force coefficient which depends on the bed roughness characteristics. \(F_e\) is the pressure force on the expansion side walls and can be defined as \(F_e = \gamma (b_2 - b_1) y_1^2\). By substituting the above mentioned relations in Eq. (14) and simplifying, the following equation can be obtained:

\[
D_{BR} \cdot (2F_{r1} \gamma + 1 - \varepsilon) + e^2 (1 - B) + 2F_{r1} \gamma B^2 = 0
\] (15)

In which \(D_{BR} = \frac{\gamma y_1}{y_1}, B = \frac{b_1}{b_2}, e = \frac{y_e}{y_1}\). Since this study is focused on S-type jump of which the toe is located at the expansion section, \(e=1\) and therefore, for S-jump on sudden expansion the following equation can be developed:

Fig. 2. Sketch of the hydraulic jump in abruptly expanded channel with rough bed
\[D_{br}^{b} - D_{br}^{r} \left[ B(2F_{ij} - \varepsilon) + 1 \right] + 22F_{ij}^2 B^2 = 0 \] (16)

This equation is similar to Eq. (2) developed for S jump by Matin et al. (1997). The difference between the two equations lies in the effect of bed roughness which is considered in the new proposed equation.

3. Experimental program

The purpose of this study is to investigate the S-jump on roughened bed under 4 expansion ratios of B=0.33, 0.5, 0.67 and 1. To this end experimental tests were performed using a rectangular laboratory canal at the Hydraulic Laboratory of Shahid Chamran University. The experiments were conducted in a 12 m long flume, 80 cm wide and 70 cm deep. The main flume width was reduced by boxes of 80 cm length and 60 cm height and different widths to 4 expansion ratios. The hydraulic jump was formed by opening the upstream gate at the desired value and the tailgate which was used to increase the downstream flow depth in order to control the toe position of the jump to occur at the expansion section. The rough elements were glued on the flume bed downstream of the expansion section in such a way that the crest of the elements was at the same level as the upstream bed. This means that the elements do not act as blocks and are not directly subject to the incoming jet. Fig. 3 shows a plan of S-jump and arrangement of the elements. Two types of expansion reaches can be created: 1) symmetric expansion in which the canal width is reduced from the both sides and 2) asymmetric expansion in which the expansion is formed by reducing the canal width from one side. In this study the expansion is of the symmetric type. The experimental procedure began by opening the upstream gate in a desired value. A special device was glued to the upstream wall of the gate so that the upstream flow depth was equal to the gate opening and the flow contraction was not observed. Then the pump was started and by opening the valve, the flow discharge was increased to reach the desired discharge. Then the tailgate was adjusted in such a way that the toe position occurred in the expansion section. This situation was kept constant for enough time to take the required data. Water depths were measured by point gauges.

Fig. 3. Plan view of the S-jump
4. Results

4.1. Shear stress coefficient

To show the effect of rough elements on shear stress coefficient, $\varepsilon$ was calculated from Eq. (16) using the experimental data. The calculated $\varepsilon$ was plotted versus Froude number which is shown in Fig. (4). Results show that the $\varepsilon$ coefficient depends on both B and $F_r^1$. For the comparison, the $\varepsilon$ coefficient obtained from Eq. (11) presented by Rajaratnam (1968), Eq. (13) presented by Izadjoo and Shafai Bejestan (2007) and for classical jump were also plotted in Fig. 4. As shown in this figure, shear stress coefficient increases as the Froude number increases. It is clear that it takes the lowest value at any Froude number for the smooth bed (classical jump). For a low Froude number eddies are small and the effect of roughness in dissipating the energy is low. As can be seen, for $F_r^1$ less than about 4 the shear coefficient for all basins are almost the same. As the Froude number increases, the effect of roughness on increasing the shear coefficient increases which is due to the creation of large eddies within the jump body. For the case of expansion jump, it is evident that the $\varepsilon$ coefficient depends on the expansion ratio. For low expansion ratios, creation of eddies increases and $\varepsilon$ is increased. The average amount of $\varepsilon$ coefficient was determined to be equal to 29.76, 8.85, 5.88, and 2.51 for B=0.33, 0.5, 0.67, and 1 in smooth beds, respectively. The $\varepsilon$ values for B=0.33 were highest for all Froude numbers tested in this study, which shows that the roughened bed in the lowest expansion ratio creates higher turbulences and greater forces. Using our experimental data and applying SPSS software, the following relation was developed for predicting the shear force coefficient for sudden expansion jump:

$$\varepsilon = 0.072 F_{r_1}^{3.103} + \ln(B^{-3.076}); R^2 = 0.88$$ (17)

4.2 Sequent depths ratio

Variation of the sequent depths ratio with approaching flow Froude number for different expansion ratios is shown in Fig. 5. This figure shows that the sequent depths ratio in roughened bed increases with expansion ratio. The line in this figure shows the sequent depths ratio for classical jump based on Blanger equation. As it can be seen, all the experimental data are below the line which verifies that the sequent depths ratio in roughened bed sudden expansion decreases.
For further discussion, the sequent depths ratio of the jump in the sudden expansion was calculated for smooth bed using equations developed by Herbrand (1973), Matin (1997) and Alhamid (2004). Then the average percentage reduction of sequent depths ratio in roughened bed sudden expansion was calculated. Table (1) shows the results. As it can be seen, the percentage reduction for the roughened bed is always greater than the smooth bed. The highest value is obtained for the expansion ratio of 0.33 which is equal to 52.9 comparing to 40.8 for the smooth bed.

### 4.3 Jump Efficiency

The relative energy dissipation or efficiency, $\eta$ is defined as the energy head loss defined as:

$$\Delta H = H_1 - H_2$$

In which $H_1$ and $H_2$ are the total energies upstream and downstream of the jump, respectively, divided by $H_1$. Fig. 6 shows $\eta$ for different expansion ratios on roughened bed. In this figure, the efficiency on roughened bed is compared to the efficiency of classical jump.

![Graph of sequent depths ratio ($D_{BR}$) versus $Fr_1$ for different expansion ratios in comparison to Blanger equation.](image)

Table 1. Percentage reduction of sequent depths ratio in smooth and roughened bed sudden expansion jump

<table>
<thead>
<tr>
<th>Study</th>
<th>Expansion ratio(B)</th>
<th>Type bed</th>
<th>$(1-D/D^*)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Herbrand(1973)</td>
<td>0.33</td>
<td>Smooth</td>
<td>43.0</td>
</tr>
<tr>
<td>Matin(1997)</td>
<td>0.33</td>
<td>Smooth</td>
<td>40.8</td>
</tr>
<tr>
<td>Alhamid(2004)</td>
<td>0.33</td>
<td>Smooth</td>
<td>39.3</td>
</tr>
<tr>
<td>This study</td>
<td>0.33</td>
<td>Rough</td>
<td>52.9</td>
</tr>
<tr>
<td>Herbrand(1973)</td>
<td>0.50</td>
<td>Smooth</td>
<td>29.0</td>
</tr>
<tr>
<td>Matin(1997)</td>
<td>0.50</td>
<td>Smooth</td>
<td>26.7</td>
</tr>
<tr>
<td>Alhamid(2004)</td>
<td>0.50</td>
<td>Smooth</td>
<td>22</td>
</tr>
<tr>
<td>This study</td>
<td>0.50</td>
<td>Rough</td>
<td>32.9</td>
</tr>
<tr>
<td>Herbrand(1973)</td>
<td>0.67</td>
<td>Smooth</td>
<td>18.0</td>
</tr>
<tr>
<td>Matin(1997)</td>
<td>0.67</td>
<td>Smooth</td>
<td>17.3</td>
</tr>
<tr>
<td>Alhamid(2004)</td>
<td>0.67</td>
<td>Smooth</td>
<td>12.5</td>
</tr>
<tr>
<td>This study</td>
<td>0.67</td>
<td>Rough</td>
<td>25</td>
</tr>
<tr>
<td>Herbrand(1973)</td>
<td>1</td>
<td>Smooth</td>
<td>0</td>
</tr>
<tr>
<td>Matin(1997)</td>
<td>1</td>
<td>Smooth</td>
<td>0</td>
</tr>
<tr>
<td>Alhamid(2004)</td>
<td>1</td>
<td>Smooth</td>
<td>0</td>
</tr>
<tr>
<td>This study</td>
<td>1</td>
<td>Rough</td>
<td>15.9</td>
</tr>
</tbody>
</table>
As it can be seen from this figure, the S-jump in abruptly expanded channel on roughened bed is more efficient than the same jump on smooth bed and the classical jump. Also, the efficiency is increased by decreasing B. This is, as previously discussed, due to creation of the large eddies between roughness elements.

The calculated jump efficiency for S-jump on roughened bed compared to S-jump on smooth bed and classical jump is presented in Table (2). The efficiency increases as the expansion ratio decreases and for the jump on roughened bed it takes a higher value. For example, the average percentage efficiency on rough bed is about 78 percent comparing to 56% for the classical jump and 73% on smooth bed with sudden expansion. In average, the existing roughness in abrupt expansion increases the efficiency about 5% and 20% into the same values of smooth S-jump and classical jump, respectively.

![Graph showing the relationship between η and Fr1 for jump on both smooth and roughened bed sudden expansion jump](image)

**Table 2- Efficiency percentages of hydraulic jump in different jumps**

<table>
<thead>
<tr>
<th>Study</th>
<th>Expansion</th>
<th>Type</th>
<th>% η</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classical jump</td>
<td>0.33</td>
<td>Smooth</td>
<td>56</td>
</tr>
<tr>
<td>Matin(1997)</td>
<td>0.33</td>
<td>Smooth</td>
<td>73</td>
</tr>
<tr>
<td>Alhamid(2004)</td>
<td>0.33</td>
<td>Smooth</td>
<td>72</td>
</tr>
<tr>
<td>This study</td>
<td>0.33</td>
<td>Rough</td>
<td>78</td>
</tr>
<tr>
<td>Classical jump</td>
<td>0.50</td>
<td>Smooth</td>
<td>41</td>
</tr>
<tr>
<td>Matin(1997)</td>
<td>0.50</td>
<td>Smooth</td>
<td>56</td>
</tr>
<tr>
<td>Alhamid(2004)</td>
<td>0.50</td>
<td>Smooth</td>
<td>53</td>
</tr>
<tr>
<td>This study</td>
<td>0.50</td>
<td>Rough</td>
<td>61</td>
</tr>
<tr>
<td>Classical jump</td>
<td>0.67</td>
<td>Smooth</td>
<td>37</td>
</tr>
<tr>
<td>Matin(1997)</td>
<td>0.67</td>
<td>Smooth</td>
<td>47</td>
</tr>
<tr>
<td>Alhamid(2004)</td>
<td>0.67</td>
<td>Smooth</td>
<td>44</td>
</tr>
<tr>
<td>This study</td>
<td>0.67</td>
<td>Rough</td>
<td>54</td>
</tr>
<tr>
<td>Classical jump</td>
<td>1</td>
<td>Smooth</td>
<td>29</td>
</tr>
<tr>
<td>Matin(1997)</td>
<td>1</td>
<td>Smooth</td>
<td>35</td>
</tr>
<tr>
<td>Alhamid(2004)</td>
<td>1</td>
<td>Smooth</td>
<td>33</td>
</tr>
<tr>
<td>This study</td>
<td>1</td>
<td>Rough</td>
<td>44</td>
</tr>
</tbody>
</table>
4.4. Toe Parameter

This parameter indicates the sensitivity of the toe displacement due to tail water variations. The parameter can be calculated using Eq. (7) for all types of jumps on abrupt expansion and for T-jump using Eq. (8) proposed by Bremen and Hager (1993). Experimental data of this study was also applied to calculate the sensitivity parameter for S-jump on roughened bed of abrupt expansion using Eq. (7) and Eq. (8). Since for S-jump \( x_1 \to 0 \), according to Eq. 8 this parameter is only a function of \( B \) and for any expansion ratio its value is maximum comparing to the other types of jumps in expanded channel. Bremen and Hager (1993) showed that jumps in abrupt expansions with \( B<0.5 \) may become strongly asymmetric if the toe is sufficiently close to the expansion section which forms S-jump. Sensitivity ratio was computed using Eq. 7 and then plotted versus Froude number which is shown in Fig. 7. Although the experimental data show some dispersion, the average sensitivity number was determined to be 0.22, 0.32, 0.41 and 0.6 for expansion ratios of 1, 0.67, 0.5 and 0.33, respectively. This parameter was calculated from Bremen and Hager (1993) to be in the order of 0.0, 0.18, 0.29 and 0.43, respectively. Results indicated that the maximum sensitivity parameter is obtained for the expansion ratio of 0.33 and also an increase of about 40% occurs comparing to the same value for S-jump on smooth bed of abrupt expansion. A higher value of sensitivity parameter will result in a stronger and more efficient jump.

![Fig. 7. Sensitivity parameter calculated from Eq. (7) versus Froude number](image)

Conclusions

In this study, characteristics of S-jump on roughened beds under different abrupt expansion ratios were experimentally investigated and the results were compared to the results of other researchers previously investigated for smooth bed of sudden expansion. From the analysis of the results, the following conclusions could be drawn:

- By considering the shear and friction forces in momentum equation, a new relation for predicting the sequent depths ratio on roughened bed of sudden expansion jump was developed.
- It was found that the shear force coefficient can increase up to 29.8% for jump on roughened bed sudden expansion more than jump on smooth bed.
- The sequent depths of S-jump over roughened bed can be decreased up to 20% in comparison to the case of smooth bed.
- The efficiency of S-jump with roughened bed is more than S-jump on smooth bed and classical jump. For S-jump on roughened bed the efficiency can increase up to 20% comparing to the classical jump and up to 10% comparing to the S-jump on smooth bed.

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References