

Comparison of Binomial and Power Equations in Radial Non-Darcy Flows in Coarse Porous Media

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ABSTRACT

Analysis of non-laminar flows in coarse alluvial beds has a wide range of applications in various civil engineering, oil and gas, and geology problems. Darcy equation is not valid to analyze transient and turbulent flows, so non-linear equations should be applied. Non-linear equations are classified into power and binomial equations. Binomial equation is more accurate in a wide range of velocity changes in comparison to power equation and its validity has been verified by dimensional analysis and Navier–Stokes equations. But since velocity changes are rather limited in engineering problems, power equation would be accurate enough. Non-Darcy flow analysis for the cases in which streamlines are almost parallel has been investigated by numerous investigators in pressured and free surface conditions. Radial flows are accompanied by streamlines contraction. Contracted streamlines in free-surface radial flows result in flow inflation, i.e. flow depth through the path increases significantly in comparison to parallel flows. This phenomenon makes free surface radial flows behave completely different from other types of flows. To investigate the behavior of free-surface radial flows in coarse porous media, power and binomial equations are analyzed in this paper. Furthermore, several experiments have been conducted by setting up a semi-cylindrical experimental device with a diameter and height of 6 and 3 meters, respectively. Results indicate that free-surface radial flows behave different from pressured radial flows and Non-Darcy flows in which streamlines are relatively parallel.

Keywords

Course porous media; converging flow; Radial; Non-Darcy flow

1. Introduction

Investigating flow in coarse porous media has become a popular field of study in recent years due to its wide application in civil engineering, oil and gas, geology, and other related fields. Non-linear flows through coarse porous media can be divided into two main categories. In the first category,

streamlines are almost parallel so that there is no curvature or contraction of streamlines in the plan view. This type of flow is found in both pressurized and free-surface modes. Flows through granular confined alluvial aquifers and detention dams are classified in this category. Many researchers have investigated parallel flows through coarse

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porous media in previous decades. Based on experimental investigations, researchers like Scheidegger (1963); Ward (1964); Dudgeon (1966); Ahmed and Sunada (1966); Hansen et al. (1994); Wilkins (1956); Li et al. (1998); Martins (1990); Bazargan and Shoaie (2006) and Bazargan and Zamanisabzi (2011) have developed equations for both linear and non-linear flows. Venkataraman and Rao (1998) introduced a diagram similar to Moody diagram for pipe flows, representing the variations of friction coefficient (f_k) and dimensionless Reynolds number (R_k) in coarse porous media.

In the second category of Non-Darcy flows, streamlines are contracted during the path and are known as converging flows. Like parallel flows, these flows are found in pressurized or free-surface modes. Flow through gravel filters used in water treatment plants is an example of pressurized converging flows. Other researchers like Right (1958); MacCorquodale (1970); Nasser (1970); Reddy (2006); Thiruvengadam and Kumar (1977); Reddy and Mohan (2006) and Dudgeon (1967) performed experiments to find effective parameters on pressurized converging flows. They believed that flow convergence played an important role in variation of “b” coefficient in binomial equation. Venkataraman and Rao (2000) having validated the Forchheimer law for flow through coarse porous media with converging boundaries proposed coefficients to modify a and b parameters in binomial equation. Various equations to analyze pressurized non-linear parallel and converging flows are presented so far and the most significant equations are summarized in continue.

2. Applied Equations to Analyze Non-Darcy Parallel Flows

2. 1. Power Equation

Power equation is presented as Missbach equation:

$$i = mv^n \quad (1)$$

Where v is the mean flow velocity, i is hydraulic gradient, and m and n are constants which are functions of fluid and porous media characteristics such as grading, roughness, shape and porosity.

2. 2. Binomial Equation

Proposed by Forchheimer, binomial equation is as follows:

$$i = av + bv^2 \quad (2)$$

Where v and i are the same as in Equation (1), and a and b are functions of physical characteristics of the media and fluid.

Ward (1964) and Ahmed et al. (1969) purposed the following equations to describe a and b parameters using dimensional analysis and Navier–Stokes equations, respectively.

$$a = \frac{\nu}{gk} \quad (3)$$

$$b = \frac{c_w}{g\sqrt{k}} \quad (4)$$

$$k = cd^2 \quad (5)$$

Where ν is kinematic viscosity of the fluid (m^2/s), k is the intrinsic permeability of the particles (m^2), g is acceleration of gravity (m/s^2); C_w is a constant which is a function of media characteristics such as particle size, shape, and porosity, C is non-dimensional constant of the material and d is characteristic length parameter (m) which

is an indicator of pore effective diameter.

Using Darcy–Weisbach equation, Ward proposed Equation (6) to describe friction coefficient and Reynolds Number in coarse porous media.

$$f_k = \frac{ig\sqrt{k}}{v^2} \quad (6)$$

Where f_k is friction coefficient of porous media and \sqrt{k} is length parameter (m) in Darcy–Weisbach equation. Based on Equation (2), (3), (4), and (6):

$$f_k = \frac{1}{R_k} + C_w \quad (7)$$

Where R_k is dimensionless Reynolds number calculated by using Equation (8) as follow.

$$R_k = \frac{v\sqrt{k}}{\nu} \quad (8)$$

Having achieved parameter a from Equation (3), k could be calculated. Equation (7) is known as Ward equation in coarse porous media. Based on the assumption of flow similarity through porous media with pressurized flow in pipes, Stephenson (1969) employed Darcy–Weisbach equation to determine hydraulic gradient in coarse porous media and concluded:

$$i = f_{st} \frac{v^2}{gn^2D} \quad (9)$$

$$f_{st} = \frac{800}{R_e} + f_t \quad (10)$$

$$R_e = \rho v D / \nu \quad (11)$$

Where v is average seepage velocity (m/s), g is acceleration of gravity (m/s^2), n is porosity; D is length characteristic (m) which

is usually considered as particle’s mean diameter. f_{st} is Stephenson friction coefficient and is defined by Equation (10), where Re is dimensionless Reynolds Number calculated by Equation (11). The value of f_t in Equation (10) is $f_t \approx 1$ for smooth and polished particles, $f_t \approx 2$ for semi-crushed particles and $f_t \approx 4$ for rough angular (crushed) particles.

3. Study on the Equations used to analyze pressurized Non-Darcy radial flows

3.1. Power Equation

In this form m and n are assumed to be determined by characteristics of the porous media and are independent of hydraulic properties of the flow. Figure 1 shows a schematic view of the experimental setup for pressurized radial flow permeameter. Pressurized radial flow parameters are introduced in Figure 2. In this figure, P_1 and P_2 are piezometric pressures at radius R_1 and R_2 , respectively, Θ is convergence angle, W is flow depth and dh_1 is the energy loss along a strip of porous media with a radius of dR .

Based on Power Equation the following equations may be developed for a given strip of the porous media with a thickness of dR .

$$dh_1 = m \left(\frac{Q}{R\Theta W} \right)^n dR \quad (12)$$

$$dh_1 = m \left(\frac{Q}{\Theta W} \right)^n \left(\frac{1}{R} \right)^n dR \quad (13)$$

Q is flow rate and $\frac{Q}{R\Theta W}$ is flow velocity at radius R . By integrating the right hand side between R_1 and R_2 and integrating the left hand side of Equation (13) between h_1 to h_2 : Where 1 and 2 subscripts correspond to the value of the parameters at radius R_1 and R_2 , respectively:

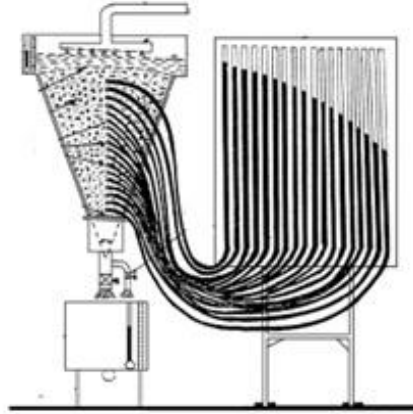


Fig. 1. Schematic view of the pressurized radial flow permeameter, Reddy, N.B. (2006)

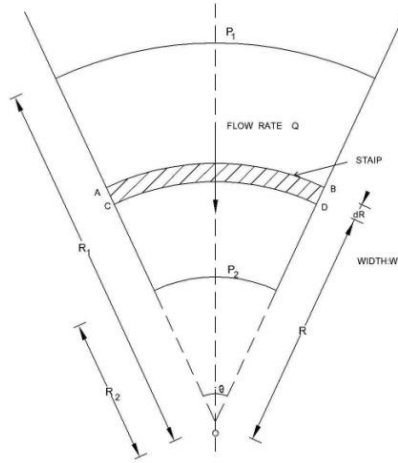


Fig. 2. Parameters of pressurized radial flow, Venkataraman, P. and Roma Mohan Rao, P. (2000)

$$\int_{h_1}^{h_2} dh_1 = m \left(\frac{Q}{\theta W} \right)^n \int_{R_1}^{R_2} \left(\frac{1}{R} \right)^n dR \quad (14)$$

Since $i = \frac{h_2 - h_1}{R_2 - R_1}$, then:

$$h_2 - h_1 = m \left(\frac{Q}{\theta W} \right)^n \frac{\left(\frac{1}{R_1} \right)^{n-1} - \left(\frac{1}{R_2} \right)^{n-1}}{n-1} \quad (15)$$

$$i = m \left(\frac{Q}{\theta W} \right)^n \frac{\left(\frac{1}{R_1} \right)^{n-1} - \left(\frac{1}{R_2} \right)^{n-1}}{(n-1)(R_1 - R_2)} \quad (17)$$

Dividing both sides of Equation (15) by $(R_2 - R_1)$:

Dividing and multiplying the right side of Equation (17) by $(R_{ave})^n$:

$$\frac{h_2 - h_1}{R_2 - R_1} = m \left(\frac{Q}{\theta W} \right)^n \frac{\left(\frac{1}{R_1} \right)^{n-1} - \left(\frac{1}{R_2} \right)^{n-1}}{(n-1)(R_2 - R_1)} \quad (16)$$

$$i = m \left(\frac{Q}{\theta R_{ave} W} \right)^n \frac{\left[\left(\frac{1}{R_1} \right)^{n-1} - \left(\frac{1}{R_2} \right)^{n-1} \right] (R_{ave})^n}{(n-1)(R_2 - R_1)} \quad (18)$$

Considering $R_{ave} = \frac{R_2 + R_1}{2}$, then:

$$i = m \left(\frac{Q}{\theta R_{ave} W} \right)^n \left[\left(\frac{1}{R_1} \right)^{n-1} - \left(\frac{1}{R_2} \right)^{n-1} \right] (R_2 + R_1)^n \quad (19)$$

$$\frac{2^n (n-1) (R_2 - R_1)}{2^n (n-1) (R_2 - R_1)}$$

Since $v = \frac{Q}{\theta R_{ave} W}$, then:

$$i = m \left[\frac{\left[\left(\frac{1}{R_1} \right)^{n-1} - \left(\frac{1}{R_2} \right)^{n-1} \right] (R_2 + R_1)^n}{2^n (n-1) (R_2 - R_1)} \right] v^n \quad (20)$$

As can be seen from Equation (20), power of v , i.e. n remains unchanged, while m is affected by a coefficient which is a function of media geometry and n parameter. Further investigations revealed that the aforementioned coefficient value for common values of R_1 , R_2 , and n is close to 1. That makes it possible to use Power Equation for both parallel and radial flows, equally. Table 1 shows this for experiments performed by Reddy (2006). As can be seen, the term in brackets in Equation (20) is close to 1 corresponding to different values of n .

3.2. Binomial Equation

Venkataraman and Rao (2000) investigated pressurized radial flows through coarse porous media. Because values of a and b parameters in Forchheimer equation, like m and n in power equation for parallel flows are functions of porous media characteristics, Venkataraman and Rao found that a and b calculated values in each experiment were different from the previous experimental results. So they looked for a solution to

reduce changes of these parameters. As a result, they proposed a binomial equation with modified coefficients (Venkataraman P. and Roma Mohan Rao P. 2000). By applying these modified coefficients a good agreement was achieved between theoretical and experimental data. Using the following equation and considering Figure 1, Venkataraman and Rao extracted C_1 and C_2 to modify a and b parameters (coefficients of Forchheimer equation for parallel flows) as Equation (24) and (25).

Assuming P_1 and P_2 as piezometric head for R_1 and R_2 , respectively (Figure 1), a_c and b_c coefficients were obtained for a and b in converging flows. In these equations, Θ is convergence angle, W is flow depth, and dh_1 is energy loss through a strip of porous media with a radius of dR . 1 and 2 subscripts correspond to parameters at R_1 and R_2 , respectively. Q is flow discharge and $\frac{Q}{R\theta W}$ is flow velocity at radius R .

$$dh_1 = \left\{ a \frac{Q}{R\theta W} + b \frac{Q^2}{R^2 \theta^2 W^2} \right\} dR \quad (21)$$

By integrating the right hand side for R_1 to R_2 and integration of the left hand side of Equation (21) for h_1 to h_2 :

$$h_2 - h_1 = \frac{Q}{\theta W} \left[\left\{ a \ln R \right\}_{R_2}^{R_1} + \frac{bQ}{\theta W} \left\{ -R^{-1} \right\}_{R_2}^{R_1} \right] \quad (22)$$

$$h_2 - h_1 = \frac{Q}{\theta W} \left[a \ln \left\{ \frac{R_1}{R_2} \right\} + \frac{bQ}{\theta W} \left\{ \frac{R_1 - R_2}{R_1 R_2} \right\} \right] \quad (23)$$

Dividing both sides of Equation (23) by $(R_2 - R_1)$, hydraulic gradient (i_c) of the flow between P_1 and P_2 is achieved in porous media with converging boundaries:

$$i_c = a_c v_1 + b_c v_1^2 \quad (24)$$

Table. 1. The coefficient of m in power equation of Equation (20) corresponding to different values of parameter n for experiments performed by Reddy (2006).

Coefficient for m in Power Equation						Coefficient for m in Power Equation					
R1(m)	R2(m)	n=1.25	n=1.50	n=1.75	n=2.00	R1(m)	R2(m)	n=1.25	n=1.50	n=1.75	n=2.00
0.100	0.050	1.056	1.076	1.099	1.125	0.800	0.750	1.008	1.008	1.009	1.010
0.150	0.100	1.019	1.026	1.033	1.042	0.850	0.800	1.009	1.010	1.010	1.011
0.200	0.150	1.010	1.013	1.017	1.021	0.900	0.850	1.010	1.011	1.012	1.012
0.250	0.200	1.006	1.008	1.010	1.013	0.950	0.900	1.012	1.012	1.013	1.013
0.300	0.250	1.004	1.005	1.007	1.008	1.000	0.950	1.013	1.013	1.014	1.015
0.350	0.300	1.003	1.004	1.005	1.006	1.050	1.000	1.014	1.014	1.015	1.016
0.400	0.350	1.002	1.003	1.004	1.004	1.100	1.050	1.015	1.015	1.016	1.017
0.450	0.400	1.002	1.002	1.003	1.003	1.150	1.100	1.016	1.016	1.017	1.017
0.500	0.450	1.001	1.002	1.002	1.003	1.200	1.150	1.017	1.017	1.018	1.018
0.550	0.500	1.001	1.001	1.002	1.002	1.250	1.200	1.017	1.018	1.018	1.019
0.600	0.550	1.002	1.003	1.003	1.004	1.300	1.250	1.018	1.019	1.019	1.020
0.650	0.600	1.004	1.004	1.005	1.005	1.350	1.300	1.019	1.019	1.020	1.020
0.700	0.650	1.005	1.006	1.006	1.007	1.400	1.350	1.019	1.020	1.021	1.021
0.750	0.700	1.007	1.007	1.008	1.008	1.450	1.400	1.020	1.021	1.021	1.022

$$a_c = c_1 a \tag{25}$$

$$b_c = c_2 b \tag{26}$$

$$c_1 = \frac{(R_1/R_2) \ln(R_1/R_2)}{\{(R_1/R_2) - 1\}} \tag{27}$$

$$c_2 = \frac{R_1}{R_2} \tag{28}$$

Developing the improved coefficients, Rao and Venkataraman only considered flow entrance velocity of each interval (V_1) in Forchheimer equation. This approach, the authors believe, may lead to considerable amount of uncertainty and approximation. They believed if mean values of piezometric heads in both sides of the interval are used to calculate the hydraulic gradient, then the mean velocity in the same interval should be used while working with Forchheimer equation. Considering the mean velocity and following the mentioned sequence, C_1 and C_2 may be calculated by Equation (29) and (30).

$$c_1 = \ln(R_1/R_2) \frac{(R_1 + R_2)}{2(R_1 - R_2)} \tag{29}$$

$$c_2 = \frac{(R_1 + R_2)^2}{4R_1R_2} \tag{30}$$

4. Study on the Equations used to analyze free-surface Non-Darcy radial flows

This paper is the product of an extensive experimental study on free-surface radial flows. A flow permeameter was established in hydraulic laboratory of Bu-Ali Sina University, Hamedan, Iran. The model was semi-cylindrical with a diameter of 6 meters and a height of 3 meters. Model dimensions were selected such that size reduction effect that usually exists in experimental models due to financial and space limitations did not exist (Figures 3 and 4). Piezometric grids were implanted within the media to measure piezometric head. To increase the accuracy of piezometric head monitoring, piezometer grids were installed on five equally-spaced vertical sheets. Totally 210 piezometers were implanted on sheets to monitor piezometric head distribution.

Applying the same analytical approach used in pressurized radial flows to analyze free-surface non-Darcy flow through coarse porous



Fig. 3. Schematic of the permeameter.

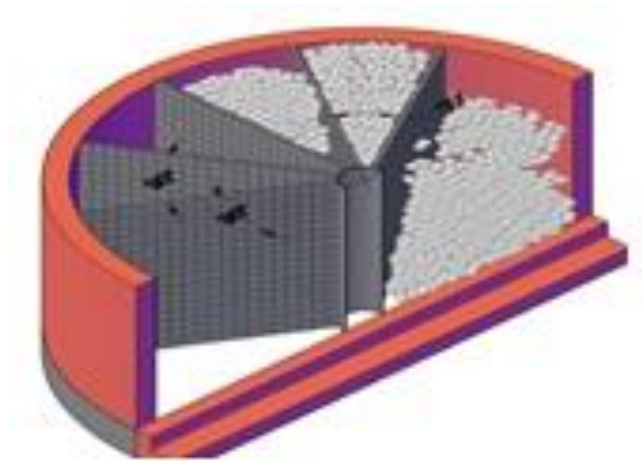


Fig. 4. Inside view of the permeameter.

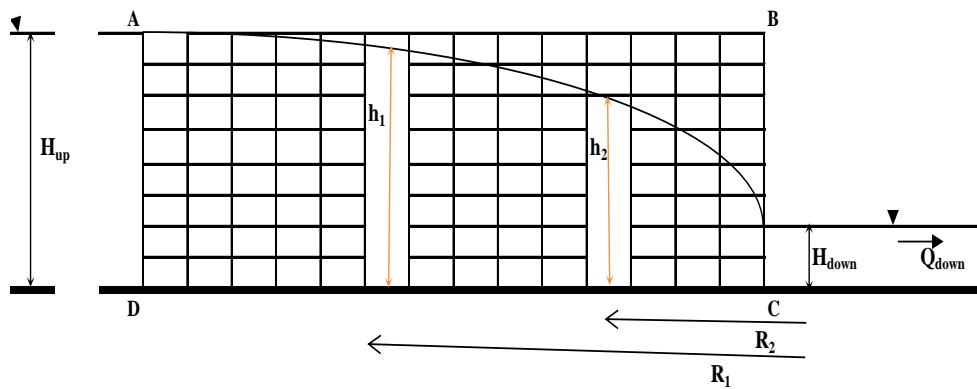


Fig. 5. Longitudinal profile of a free-surface radial flow presenting parameters influencing a and b factors in quadratic and m and n in power law equations.

media, and considering P_1 and P_2 as piezometric heads at R_1 and R_2 , respectively (Figures 5), the following equations are developed to describe a_{cf} and b_{cf} coefficients instead of a and b for converging non-linear flows in coarse porous media with free surface. In these equations, θ is convergence angle, h is flow depth at radius R , and dh_1 is energy loss along a strip of porous media with a radius of dR . Q is flow discharge and $\frac{Q}{R\theta W}$ is flow velocity at radius R .

$$dh_1 = \left(a \frac{Q}{\theta R h} + b \frac{Q^2}{\theta^2 R^2 h^2} \right) dR \quad (33)$$

$$h^2 dh_1 = Q \left(a \frac{h}{\theta R} + b \frac{Q}{\theta^2 R^2} \right) dR \quad (34)$$

Since the flow depth (h) is a function of water surface, and based on the fact that longitudinal profile of water surface usually has a parabolic shape, so h could be written as a function of R as follows.

$$h = AR^2 + BR + C \quad (35)$$

Where A , B , and C are coefficients of the parabolic equation. Substituting Equation (35) in Equation (34):

$$h^2 dh_1 = \frac{Q}{\theta} \left(a \left(AR + B + \frac{C}{R} \right) + b \frac{Q}{\theta R^2} \right) dR \quad (36)$$

Energy loss may be calculated within this interval by integrating between R_1 to R_2 in which flow depth varies from h_1 to h_2 . Subscripts 1 and 2 correspond to the parameter values at R_1 and R_2 , respectively.

$$\int_{h_2}^{h_1} h^2 dh_1 = \int_{R_2}^{R_1} \frac{Q}{\theta} \left(a \left(AR + B + \frac{C}{R} \right) + b \frac{Q}{\theta R^2} \right) dR \quad (37)$$

$$\left(h_1^3 - h_2^3 \right) / 3 = \frac{Q}{\theta} \left\{ a \left[\left(A \frac{R_1^2 - R_2^2}{2} + B(R_1 - R_2) + C \ln \left(\frac{R_1}{R_2} \right) \right) + b \frac{Q}{\theta} \left(\frac{R_1 - R_2}{R_1 R_2} \right) \right] \right\} \quad (38)$$

$$(h_2 - h_1) = \frac{Q}{\theta} * \left\{ a \left[\left(A \frac{R_1^2 - R_2^2}{2} + B(R_1 - R_2) + C \ln \left(\frac{R_1}{R_2} \right) \right) + b \frac{Q}{\theta} \left(\frac{R_1 - R_2}{R_1 R_2} \right) \right] \right\} \quad (39)$$

$$\frac{3}{h_1^2 + h_1 h_2 + h_2^2}$$

Dividing both sides of Equation (39) by $(R_1 - R_2)$, hydraulic gradient (i_{cf}) of free-surface flow in converged porous media is obtained between P_1 and P_2 .

$$i_{cf} = \frac{Q}{\theta} \left\{ a \left[\left(A \frac{R_1 + R_2}{2} + B + \frac{C \ln \left(\frac{R_1}{R_2} \right)}{R_2 - R_1} \right) + b \frac{Q}{\theta R_1 R_2} \right] \right\} \quad (40)$$

$$\frac{3}{h_1^2 + h_1 h_2 + h_2^2}$$

By defining V_{ave} , \bar{h} , and \bar{R} , as:

$$\frac{R_1 + R_2}{2} = \bar{R} \quad (41)$$

$$\frac{h_1 + h_2}{2} = \bar{h} \quad (42)$$

$$\frac{Q}{\theta \bar{R} \bar{h}} = V_{ave} \quad (43)$$

Substituting Equation (41), (42), and (43) into Equation (40):

$$i_{cf} = a \left\{ \left[\left(A \left(\bar{R}^2 \bar{h} \right) + B \bar{R} \bar{h} \right) + \frac{C \ln \left(\frac{R_1}{R_2} \right)}{\bar{R} \bar{h}} \right] \left(\frac{3}{h_1^2 + h_1 h_2 + h_2^2} \right) \right\} \quad (44)$$

$$* \left(\frac{Q}{\theta \bar{R} \bar{h}} \right) + b \left(\frac{3 \left(\bar{R}^2 \bar{h}^2 \right) / R_1 R_2}{h_1^2 + h_1 h_2 + h_2^2} \right) \left(\frac{Q}{\theta^2 \bar{R}^2 \bar{h}^2} \right)$$

$$i_{cf} = a^* \left\{ \left(A(\bar{R}^2 \bar{h}) + B\bar{R}\bar{h} + \frac{C \ln\left(\frac{R_1}{R_2}\right)}{R_1 - R_2} \bar{R}\bar{h} \right) \left(\frac{3}{h_1^2 + h_1 h_2 + h_2^2} \right) \right\}^* V_{ave} + \quad (45)$$

$$b \left(\frac{3(\bar{R}^2 \bar{h}^2)/R_1 R_2}{h_1^2 + h_1 h_2 + h_2^2} \right) V_{ave}^2 \quad (46)$$

$$a_{cf} = c_1 a \quad (47)$$

$$b_{cf} = c_2 b \quad (48)$$

$$c_1 = \left(A\bar{R} + B + C \frac{\ln\left(\frac{R_1}{R_2}\right)}{R_1 - R_2} \right) \left(\frac{3\bar{R}\bar{h}}{h_1^2 + h_1 h_2 + h_2^2} \right) \quad (49)$$

$$c_2 = \frac{3(\bar{R}^2 \bar{h}^2)/R_1 R_2}{h_1^2 + h_1 h_2 + h_2^2} \quad (50)$$

Results show that if h_1 and h_2 variations would be small from R_1 to R_2 , then Equation (49) and (50) could be replaced by Equation (29) and (30). Similarly, by ignoring h_1 and h_2 variations within the range of R_1 to R_2 , Equation (20) could be used as a power equation to analyze non-Darcy flows through coarse porous media with free-surface. The term inside brackets in Equation (20) corresponding to different values of parameter n in power equation is presented in table 2.

As can be seen, the term inside brackets in Equation (20) was found to have values close to 1.

Table 2. The term inside the brackets of Equation (20) corresponding to different values of parameter n in power equation

Coefficient for m in power Equation			
R1	R2	N=1.25	N=1.50
0.25	0.50	1.06	1.08
0.50	0.75	1.02	1.03
0.75	1.05	1.01	1.02
1.05	1.40	1.01	1.01
1.40	1.80	1.01	1.01
1.80	2.25	1.01	1.01
2.25	2.75	1.00	1.01

5. Conclusions

Flow through coarse porous media can be categorized into parallel and convergence flows. Investigations showed that flow in both categories is a non-Darcy flow and binomial and power equations are still valid. Although binomial equation is more accurate in a wide range of velocity changes in comparison to power equation, since the range of velocity changes is rather limited in engineering problems, power equation

would be of enough accuracy. So power equation is considered as the base equation in numerical models. Those models that were established based on the developed Parkin equation in Cartesian coordinates, and the model developed in this study in cylindrical coordinates all consider the power equation as the base equation. Contraction of streamlines is reported in both pressurized converging and free-surface radial flows. Contracted streamlines in radial

flows with free-surface condition lead to flow inflation and as a result, flow depth increased significantly compared to parallel flow condition. In pressurized converging flows, inflation cannot occur due to the existence of the wall margins. Compared to other types of the non-linear flows, free-surface radial flows are subjected to greater values of velocity variations to hydraulic gradient variations, and vice versa in the case of parallel and pressurized converging flows. It seems that this fact results in a significant difference in flow behavior of free-surface radial flows compared to other types of flows. More than 1500 piezometric heads were recorded during this work. Having analyzed the experimental results of this study, it was concluded that applying the initial form of power equation will lead to better results compared to Forchheimer equation. Experimental results also confirmed this analytical conclusion. Forchheimer equation could be used in these types of flows by applying modification coefficients.

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