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Mathematical Modeling of Flow in a Granular Permeable Bed Channel

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Abstract

In this research, a two-dimensional mathematical model using $k-\varepsilon$ turbulence model was developed in a rectangular channel with granular permeable bed. Finite volume method was used for numerical solution of governing equations. The equations considered are discretized using Hybrid difference scheme while applying SIMPLE algorithm to correct velocity components in the continuity equation. In the developed turbulence method, wall function was considered for simulation of the boundary conditions. The results show that the proposed model yields better values than the earlier laboratory experiments by the researchers. Gravity component is a negative source in V momentum equation. This negative source is integrated into pressure content via a tricky manner to converge the numerical solution. By various repetitions, appropriate range of "under relaxation factor" is obtained in order to make a better and faster convergence in resolving the equations. Results of the mathematical model as well as earlier laboratory results show that changing of the permeable bed conditions will affect the turbulence parameters especially in the area near the permeable boundary. Comparison of the velocity components, kinetic energy and dissipation rate of the kinetic energy which is derived from the earlier laboratory simulations as well as mathematical results indicates the precision of the proposed mathematical model.

Keywords

Wall function, Turbulence parameters, Reynolds shear stress, Kinetic energy

1. Introduction

Permeable beds in many rivers and channels cause the flow infiltration into the bed. This infiltration has an enormous effect on hydraulic conditions near the bed. Infiltration or bed suction in permeable bed channels causes a change in velocity distribution, bed shear stress and turbulent flow conditions, in comparison with the flow over a non-permeable bed. The flow over a waterway intake constructed on the river bed is a turbulent one over the permeable bed. Increased shear stress caused by suction to the bed and its effect on material movement and eventually occurrence of bed scouring at the intake proves the importance of studying this type of flow.

Some earlier laboratory and mathematical model researches were performed on flows over the permeable beds. Gupta et al. (1985) conducted laboratory studies in a free surface channel with permeable bed. Their studies were about flow velocity profile, bottom

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friction and infiltration rate over permeable bed. The investigation of laboratory results showed that a logarithmic velocity distribution could be considered for such a flow but channel bottom datum should be considered at a distance (δz) below the channel bottom as well as Von Karman's constant should be changed from 0.4 for the flow over non-permeable bed to 0.28 for a channel with permeable bed. The following relationship for velocity distribution has been provided by above mentioned research.

$$\frac{U}{u_*} = \frac{1}{\kappa} Ln\left(\frac{y+\delta z}{k_s}\right) + A \qquad (1)$$

Where U is flow velocity over permeable medium, u_* is shear velocity, κ is Von Karman constant equal to 0.28, y is distance to channel bottom, k_s is bed roughness, A is a constant and δz is distance from bottom to datum. Value of δz was equal to one-third of d_{50}. This value will cause an accurate result for the velocity from above equation.

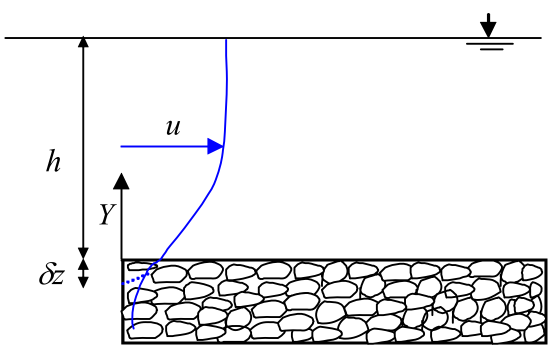

Fig.1. Velocity distribution over the bed, with bed infiltration (Gupta et al. 1985)

Eq. (1) and Fig. 1 indicate that when suction happens at the channel bed, nonslip condition is not met and velocity is not equal to zero on bed.

The researchers have found that infiltration to the bed increases the bottom shear stress (friction). Eq. (2) which is used to determine flow friction coefficient (f) for permeable materials was also derived from this laboratory study [3].

$$\frac{1}{\sqrt{f}} = \frac{1}{\sqrt{8}\kappa}\left[Ln\left(\frac{h+\delta z}{k_s}\right) - 1\right]\frac{1}{\sqrt{8}}\frac{u_s}{u_*} \qquad (2)$$

In which u_s is bed velocity and h is water depth.

Maclean (1991) conducted an experimental research on the velocity distribution in a channel with bed suction. He found that velocity distribution diverts from standard logarithmic profile and as flow progresses in suction area, this diversion will also increase. From the physical model tests he observed that by increasing the bottom suction discharge, bottom shear stress will also increase. This increased stress could cause velocity and erosion intensity and make some pits at the bed.

Prinos et al. (1998) have done experimental researches on a turbulence flow in a channel with permeable bed. They changed the porosity and thickness of permeable materials and found that by increase in porosity and thickness, normalized velocity (U/u_*) will augment. They also observed that Von Karman constant (κ) decreases in a permeable bed in comparison with a non-permeable one by drawing normalized velocity distribution in the normalized depth.

Prinos et al. (2003) have developed turbulence mathematical model for flow in an open channel with permeable bed and compared their mathematical results with results of earlier tests done on a physical model. They made the permeable medium by an artificial permeable one which consists of transverse rows of cylindrical rods placed at the channel bottom. Choosing this type of permeable medium has made it possible for researchers to measure velocity inside porous media in addition to flow over permeable medium.

Flow simulation inside porous medium was done in a microscopic level. They conducted numerical solving of Navier-Stocks equations by $k-\varepsilon$ turbulence model with low Reynolds number. They studied the change in velocity profile and turbulence parameters in the over flow and flow passes through artificial permeable medium. A summary of these researches are described below:

Permeable medium conditions, regarding structural alignment of rods have a slight effect on the flow over permeable medium near the boundary. Average velocity of a flow over permeable medium is less than the flow average velocity over a smooth and non-permeable surface. The passing capacity over permeable medium in comparison with a smooth and non-permeable surface has a less value. For such flow, A and κ constant coefficients in logarithmic velocity formula have different values in comparison with flow over smooth and non-permeable bed.

2. Mathematical modeling

In this research, mathematical simulation of flow over the granular permeable bed has been developed. For this purpose, finite volume method has been used for numerical solution of flow equations. Staggered grid has been used for the domain mesh generation. Selection of the staggered grid is to improve numerical conditions and remove oscillation of the velocity values. Velocity, pressure and also turbulence parameters including kinetic energy, rate of kinetic energy distribution and turbulence viscosity for flow over the granular (permeable) medium are given as output values in the proposed mathematical model.

Momentum equations and $k-\varepsilon$ formulas are of the same type of the convection-diffusion equations. Hybrid approximation has been used to discretize the equations. This approximation is simple and compatible with the nature of the problem. If ϕ parameter is considered as a flow property, two-dimensional general transport equations for steady state condition will be as below.

$$U\frac{\partial\phi}{\partial x}+V\frac{\partial\phi}{\partial y}=\frac{\partial}{\partial x}\left(\Gamma\frac{\partial\phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\Gamma\frac{\partial\phi}{\partial y}\right)+S \quad (3)$$

Where ϕ could be each of the velocity, kinetic energy or energy dissipation parameters and S is a source term that contains all additional terms related to the above equation. Hybrid method of the general discrete transport equation is one which has been applied by Versteeg et al. (1995).

3. Governing equations

Governing equations include U momentum, V momentum, continuity, kinetic energy and rate of energy dissipation equations defined as below:

U momentum:

$$U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=\text{g}.\sin\theta-\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{\partial}{\partial x}\left((\nu_t+\nu)\frac{\partial U}{\partial x}\right)+\frac{\partial}{\partial y}\left((\nu_t+\nu)\frac{\partial U}{\partial y}\right) \quad (4)$$

V momentum:

$$U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}=-\mathrm{g}.\cos\theta-\frac{1}{\rho}\frac{\partial P}{\partial y}+\frac{\partial}{\partial x}\left((\nu_t+\nu)\frac{\partial V}{\partial x}\right)+\frac{\partial}{\partial y}\left((\nu_t+\nu)\frac{\partial V}{\partial y}\right) \quad (5)$$

Continuity:

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0 \quad (6)$$

Kinetic energy:

$$U\frac{\partial k}{\partial x}+V\frac{\partial k}{\partial y}=\frac{\partial}{\partial x}\left((\nu_t+\nu)\frac{\partial k}{\partial x}\right)+\frac{\partial}{\partial y}\left((\nu_t+\nu)\frac{\partial k}{\partial y}\right)$$
$$+2\nu_t(\frac{\partial U}{\partial x})^2-\frac{2}{3}k\frac{\partial U}{\partial x}+\nu_t(\frac{\partial U}{\partial y})^2+\nu_t\frac{\partial V}{\partial x}.\frac{\partial U}{\partial y}-\varepsilon \quad (7)$$

Rate of energy dissipation:

$$U\frac{\partial \varepsilon}{\partial x}+V\frac{\partial \varepsilon}{\partial y}=\frac{\partial}{\partial x}\left((\nu_t+\nu)\frac{\partial \varepsilon}{\partial x}\right)+\frac{\partial}{\partial y}\left((\nu_t+\nu)\frac{\partial \varepsilon}{\partial y}\right)$$
$$+C_{\varepsilon1}\frac{\varepsilon}{k}(2\nu_t\left(\frac{\partial U}{\partial x}\right)^2-\frac{2}{3}k\frac{\partial U}{\partial x}+\nu_t\left(\frac{\partial U}{\partial y}\right)^2+\nu_t\frac{\partial V}{\partial x}.\frac{\partial U}{\partial y})$$
$$-C_{\varepsilon2}\frac{\varepsilon^2}{k} \quad (8)$$

To equilibrate the forces exerted on an element, a gravitational force term has been shown up in momentum relationships. In V momentum equation "$-g.\cos\theta$" will be as a negative source. In the first iteration of the numerical solution, the values of all U and V velocities as well as the pressure in the nodes were set to zero. This negative source has noticeable values in the discrete form of V momentum equation. This negative source produces non-real negative values for velocity in Y direction in any flow condition at first iteration and causes divergence in numerical solving process. In order to resolve the problem, a technical trick must be used to eliminate the gravity term apparently. Thus, if the pressure distribution is hydrostatic, the pressure term will be divided into a dynamic pressure and hydrostatic pressure. The process for resolving the problem is described by Fig. 2 as below:

$$P=P_s+P_d$$

$$P_s=\gamma d=\gamma(h-y)\cos\theta$$

$$\frac{\partial y}{\partial x}=-\tan\theta$$

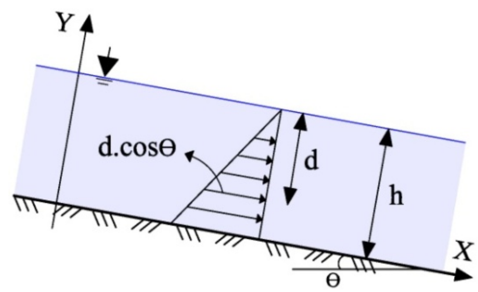

Fig. 2. Hydrostatic pressure in a channel with inclination angle of θ.

Gravity and pressure terms are combined together in U and V momentum equation:

$$\mathrm{g}.\sin\theta-\frac{1}{\rho}\frac{\partial P}{\partial x}=\mathrm{g}.\sin\theta-\frac{1}{\rho}\frac{\partial(P_s+P_d)}{\partial x}=-\frac{1}{\rho}\frac{\partial P_d}{\partial x} \quad (9)$$

$$-\mathrm{g}.\cos\theta-\frac{1}{\rho}\frac{\partial P}{\partial y}=-\mathrm{g}.\cos\theta-\frac{1}{\rho}\frac{\partial(P_s+P_d)}{\partial y}=-\frac{1}{\rho}\frac{\partial P_d}{\partial y} \quad (10)$$

As shown above, gravity term is merged in pressure term. Hydrodynamic pressure is obtained by numerical solution. Pressure gradient in Y and X direction, not pressure values in each node, is used in iteration cycles. Therefore, if there is not substantial curvature in water surface, above mention technical manner can be used to remove the problem.

Table 1. Appropriate Underrelaxation factor range for the mathematical modeling

Equation	U momentum	V momentum	Continuity	Energy	Energy dissipation rate	Turbulence viscosity
Underrelaxation factor	0.90–0.98	0.90–0.98	0.02–0.10	0.60–0.70	0.60–0.70	0.40–0.60

4. Mathematical modeling process

In this study a mathematical modeling process is applied to solve U momentum, V momentum, continuity, kinetic energy and kinetic energy dissipation equations. After the completion of mathematical solution to the equations addressed above, the iterations will be repeated till the convergence condition reaches the target value. Continuity equation shall also be satisfied as a convergence condition. In other words, in continuity. equation the normalized value of the total residual should be very small. Numerical iteration will be finished when the convergence is achieved.

Under relaxation factor is used to assure an appropriate convergence. Appropriate Under relaxation factor range used in this modeling is shown in table 1. SIMPLE algorithm has been used in order to insert continuity in numerical solution process. This algorithm modifies the values of velocities and pressure components of each node at each step of iteration. It has a way to obtain continuity.

5. Boundary conditions

Boundaries of simulation domain consist of inlet, outlet, water surface and permeable bed. It is assumed that inlet flow is a fully developed flow. This assumption was reasonable. The boundary conditions are defined as follow.

5.1. Inlet boundary conditions

First, a fully developed flow is made in a channel with impermeable bed as well as a water surface parallel to bed. This fully developed flow will be as flow at inlet boundary and hence all needed parameters are known at the inlet accordingly. Using fully developed flow at the entrance instead of a uniform flow leads to a shorter domain as well as quicker convergence than using a uniform flow.

5.2. Bed conditions

Bed is a permeable surface in a channel or stream. Infiltration into the permeable bed causes new hydraulic conditions compared with the flow on non-permeable surfaces. There is no nonslip condition at the bed as well as logarithmic profile of velocity is different from the universal logarithmic profile.

5.3. U velocity in bed

Wall function specified U velocity at bed boundary. Wall function has been used for flow over the non-permeable bed and results in appropriate U velocities in such flow. There is no evidence about using this function to reach bed boundary condition for permeable beds. Hence, using this approach for permeable surfaces is needed to be testified.

Previous research results show that velocity distribution on permeable bed is somewhat different from velocity distribution on non-permeable bed. Gupta et al. (1985), Chen et al. (2004) and Chiew et al. (2001) have pointed in their theory-laboratory researches that logarithmic distribution of Eq. (1) on passing flow on the permeable medium is correct. Flow conditions were fully developed on their researches. Shear velocity is derived from the velocity distribution.

$$u_* = \sqrt{\frac{\tau_w}{\rho}} = \frac{kU}{Ln\left(\frac{y+\delta z}{k_s}\right)+\kappa A} \quad (11)$$

Balance between energy production and dissipation of energy will also give another relationship which shear stress can be achieve using it.

$$u_* = \sqrt{\frac{\tau_w}{\rho}} = k^{0.5}.{C_\mu}^{0.25} \quad (12)$$

Then:

$$\tau_w = -\rho.k^{0.5}.{C_\mu}^{0.25}.\kappa.\frac{1}{Ln\left(\frac{y+\delta z}{k_s}\right)+\kappa A}.U \quad (13)$$

Thus, shear force is:

$$F_s = -\rho.k^{0.5}.{C_\mu}^{0.25}.\kappa.\frac{1}{Ln\left(\frac{y+\delta z}{k_s}\right)+\kappa A}.A_{cell}.U \quad (14)$$

Therefore, "source term" value in U momentum relationship will be obtained for the near bed boundary element as described below.

$$S_p = -\rho.k^{0.5}.{C_\mu}^{0.25}.\kappa.\frac{1}{Ln\left(\frac{y_p+\delta z}{k_s}\right)+\kappa A}.A_{cell} \quad (15)$$

5.4. V velocity in bed

There is a discharge into bed, so a perpendicular velocity exists at the bed. If flow over the permeable bed reaches to fully developed condition, there will be very low discharge to the bed. Therefore, velocity value is insignificant for fully developed flow. V Velocity at bed is considered as zero.

5.5. Kinetic energy (k) in bed

Versteeg et al. (1995) have obtained κ flux for a volume element closed to the permeable bed as below.

$$k\ source = (\tau_w U_p - \rho {C_\mu}^{0.75} {k_p}^{1.5} u^+)\frac{\Delta Vol}{y_p} \quad (16)$$

By substituting $u^+ = \frac{U}{u_*}$ from Eq. (1), k flux for permeable surface is obtained such as below.

$$k\ source = (\tau_w U_p - \rho C_\mu^{0.75} k_p^{1.5}(\frac{1}{\kappa} Ln\left(\frac{y_p+\delta z}{k_s}\right) + A))\frac{\Delta Vol}{y_p} \quad (17)$$

$$S = S_p k_p + S_u \quad (18)$$

$$S_p = -\frac{\rho C_\mu^{0.75} k_p^{1.5}(\frac{1}{\kappa}Ln\left(\frac{y_p+\delta z}{k_s}\right)+A)}{y_p}.\Delta Vol \quad (19)$$

$$S_u = \frac{\tau_w U_p}{y_p}\Delta Vol \quad (20)$$

5.6. Dissipation rate of Kinetic energy (ε) in bed

The value for energy distribution at the first node near the bed, ε_P is considered as a known value.

$$\epsilon = P_k$$

$$\epsilon = -\overline{uv}\frac{\partial U}{\partial y} = {u_*}^2\frac{\partial U}{\partial y} = {u_*}^2\frac{u_*}{\kappa.y} = \frac{{u_*}^3}{\kappa.y} \quad (21)$$

5.7. Water surface boundary conditions

Differentiation of all parameters in y direction is zero. It means:

$$\frac{\partial U}{\partial y} = 0\ ,\ \frac{\partial V}{\partial y} = 0\ ,\ \frac{\partial k}{\partial y} = 0\ ,\ \frac{\partial \varepsilon}{\partial y} = 0$$

6. Model verification

There are some physical laboratory records as well as some verified mathematical model results about flow over porous media, but only hydraulic and physical model data of Prinos et al. (1998) and verified mathematical results of Prinos et al. (2003) is available. Therefore, these results are used to verify accuracy of the proposed mathematical simulation model. First one is used to confirm accuracy of flow velocity profile and second one is used to verify accuracy of turbulence components, kinetic energy and Reynolds shear stress over porous bed. Prinos et al. (1998) conducted their experiments on a flume. Flume specifications are 25 centimeter width, 50 centimeter depth, 12 meter length, and 0.002 degree of longitudinal bed slope. Reportedly, bottom of the channel has been covered with aggregate materials. Porosity of this material was equal to 0.7 and its roughness was 2.9 millimeter.

Flow depth and discharge values of 0.5 cm and 6.7 lit/sec were considered, respectively. Physical data collection was done in fully developed flow zone in flume. Up to now, universal velocity distribution on the permeable beds, unlike impermeable beds, has not been found. There was a special form for each research. In order to use wall function, constants of the velocity distribution shall be specified in order to obtain the desired conditions.

Eq. (1) was used as velocity distribution on the permeable beds in Gupta et al. (1985) laboratory tests. These researchers proposed the value of δz equal to one-third of the bed roughness value. Bed roughness is considered as $k_s = 2.9$ mm in Prinos et al. (1998) laboratory research which determines the value of distance up to datum plane of $\delta z = \frac{k_s}{3} = 0.97$ mm. By using above mentioned laboratory data, U^+ versus Y^{++} curve is plotted in Fig.3. Using this figure κ and A constants are extracted as 0.249 and 8.627, respectively, where $Y^{++}$is equal to$\frac{y_p+\delta z}{k_s}$. Velocity profile on permeable bed is achieved by using the above mentioned constants:

$$\frac{U}{u_*} = \frac{1}{0.249} Ln\left(\frac{y+0.00097}{0.0029}\right) + 8.627 \tag{22}$$

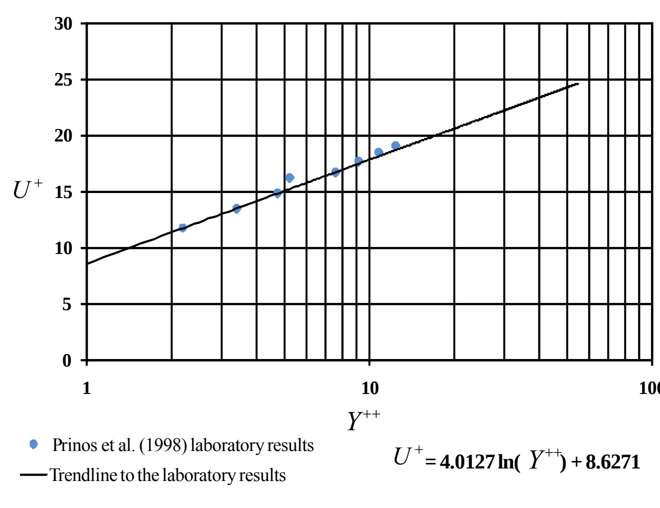

Fig. 3. Velocity distribution results obtained from Prinos et al. (1998) laboratory results.

The above velocity profile was used as wall function formula in mathematical simulation. Also Prinos et al. (1998) laboratory data were used for the mathematical simulation. U velocity results and normalized U velocities are shown in Figs. 4 and 5, respectively. A comparison between Prinos et al. (1998) laboratory data and mathematical results of this study can be seen in these figures.

It is observed that the results obtained by the mathematical model are consistent with earlier physical experimental results.

Prinos et al. (2003) developed mathematical simulation of the flow over artificial porous beds consisting of some bars as shown in Fig. 6. The researchers considered two different cases of porous beds in their study. In the first case, there is a staggered arrangement of the bars in bed and in the second case, the bed consists of a non-staggered arrangement of bars.

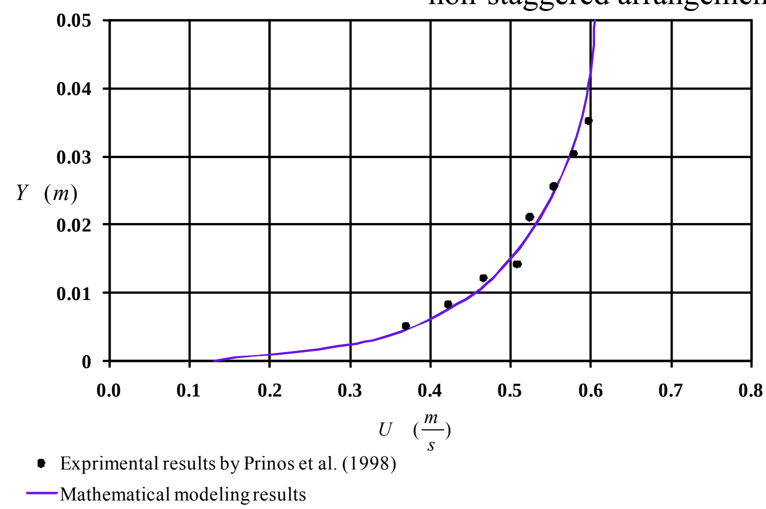

Fig.4. Comparison of the velocity profiles obtained from the laboratory and mathematical modeling.

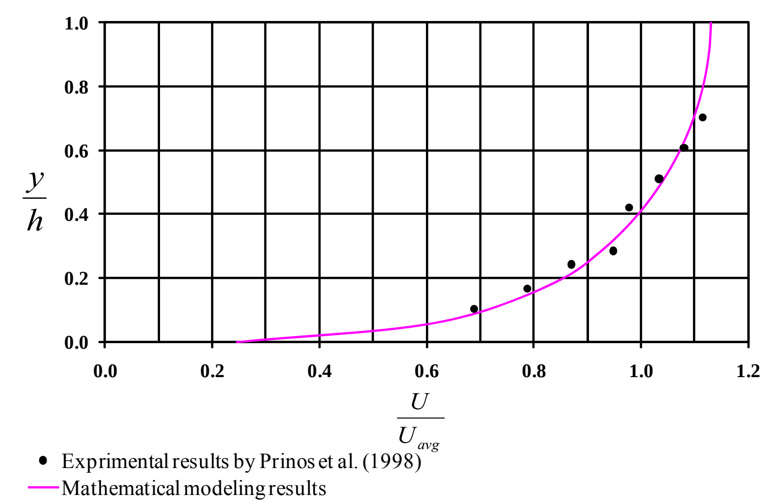

Fig.5. Comparison of the normalized velocity-depth profiles obtained from the laboratory and mathematical modeling. (U_{avg} is average velocity in the section)

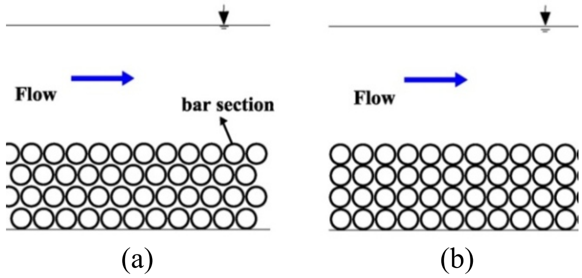

Fig.6. (a) Staggered arrangement, (b) non-staggered arrangement of bars in permeable bed

Two cases are shown in Fig. 6. Porosity of permeable bed was 0.8286. Bed slope, rod diameter and water depth of 12×10^{-3}, 11.5 mm, and 55 mm were considered, respectively.

Flow was considered as a two-dimensional flow and the model was run in a fully developed flow condition. $k-\varepsilon$ turbulence method with low Reynolds number was used. Their results were shown in normalized curves. Normalized values of kinetic energy and shear stress parameters obtained from the mathematical model are compared with their results in Figs. 7 and 8.

As shown in the figures, the turbulence parameters results were close to the obtained results using the mathematical model.

There is a little difference between the results in the area near the porous bed that can be attributed to two reasons. Firstly, permeable bed structure is effective on the turbulence parameters values especially near the bed. The difference between the results of the two staggered and non-staggered arrangements of bars in the study of the three researchers is obvious. Secondly, differences in mathematical simulation methods used can also cause the differences in the results. It is noted that differences between the obtained results is more evident for the areas adjacent to the bed.

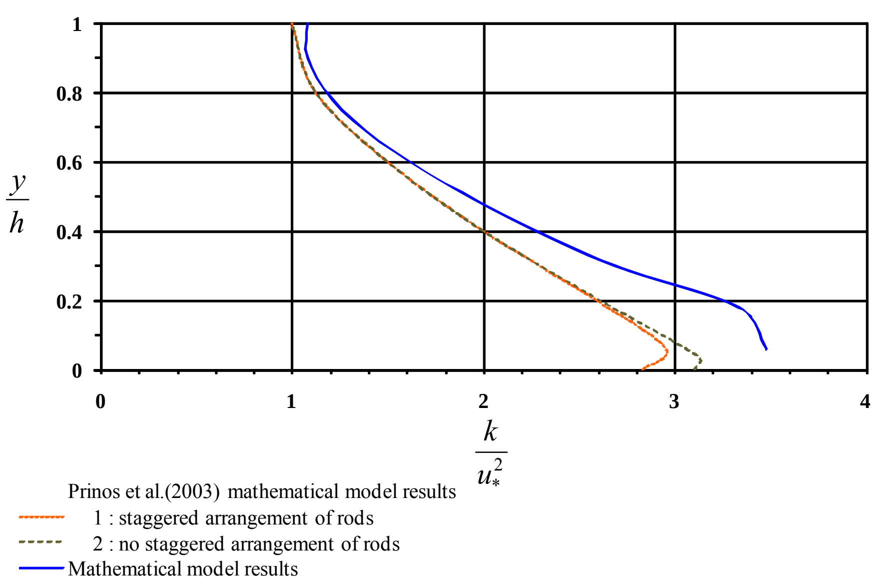

Fig. 7. Comparison of the results for normalized kinetic energy derived from current mathematical simulation with mathematical simulation results of Prinos et al. (2003).

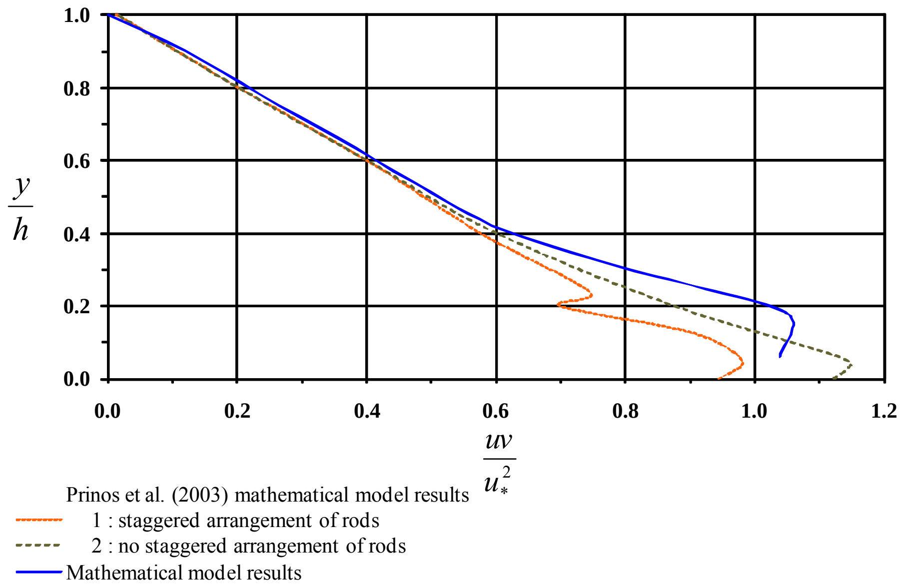

Fig. 8. Comparison of the normalized Reynolds shear stress values derived from the current mathematical simulation with mathematical simulation results of Prinos et al. (2003).

$k-\varepsilon$ model with a low Reynolds number can be an appropriate model for the flow so close to the boundary and therefore, will consider the conditions near the bed, while $k-\varepsilon$ model with wall function, which used in this simulation, is able to model the flow in distant areas from the boundary. Comparison of the obtained velocity results with laboratory results of Prinos et al. (1998), and also comparison of the kinetic energy distribution and Reynolds shear stress values with the mathematical results of Prinos et al. (2003) showed the accuracy of the mathematical model.

7. Conclusion

In this research a simulation model was developed for flow over a permeable bed channel. The results of the simulation can be summarized as follows:

-Like other studies, it can be seen that velocity distribution on the permeable bed is a logarithmic one, but it is different from the usual logarithmic velocity distribution on the impermeable bed.

-Comparison of the mathematical simulation results with the corresponding values obtained from previous investigator's laboratory-mathematical researches shows the accuracy of the proposed mathematical model.

-In order to simulate a flow over a permeable bed, it is possible to use $k-\varepsilon$ model with wall function. Although wall function does not provide the values closed to bed, but this research showed the accuracy of the proposed model for such a flow.

-Negative source in momentum equations causes a divergence in solving V momentum equation. Gravity and pressure terms merge by a technical trick to eliminate this negative source apparently.

-The model results as well as other research results show that the nature of permeable bed affects the values of the turbulence parameters. This affect is more sensible near the bed.

-Appropriate range of Under-relaxation factor in the governing equations was proposed. The proposed range may be used in future mathematical simulations by other investigators.

-Shear stress on the bed is directly related to the turbulence parameters. Therefore, shear stresses over different bed structures must be different. Energy loss of the flow is dependent upon shear stress value. Based on this fact, it is predicted that the energy loss is affected by the nature of the permeable structure.

Nomenclature

A Constant in velocity profile

$C_{\varepsilon 1}$ Constant in epsilon equation

$C_{\varepsilon 2}$ Constant in epsilon equation

C_{μ} Constant in source terms

d distance from water surface (m)

D_{50} diameter of grain material (m)

f friction coefficient

g acceleration of gravity (m/s^3)

h water depth in channel (m)

k kinetic energy (m^2/s^2)

k_s bed roughness (m)

p Pressure (N/m^2)

p_s hydrostatic pressure (N/m^2)

p_d hydrodynamic pressure (N/m^2)

u horizontal component of instantaneous velocity (m/s)

u_s slip velocity (m/s)

u_* shear velocity (m/s)

U time average horizontal component of velocity (m/s)

v vertical component of instantaneous velocity (m/s)

v_s bottom suction velocity in vertical direction (m/s)

V time average vertical velocity component of velocity (m/s)

y distance from channel bottom (m)

ε dissipation rate of kinetic energy (m^2/s^3)

φ representative of fluid property

δz parameter of length (m)

κ Von Karman's constant

κ_* Von Karman's constant for flow over permeable bed

θ angel

υ Kinematic viscosity (m^2/s)

υ_t turbulence viscosity (m^2/s)

ρ water mass density(kg/m^3)

τ_w bed shear stress (N/m^2)

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